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EFFECT OF RADIATION ON UNSTEADY MHD HEAT AND MASS TRANSFER FLOW OF A CHEMICALLY REACTING FLUID PAST AN IMPULSIVELY STARTED VERTICAL PLATE

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ABSTRACT

The interaction of free convection with thermal radiation on an unsteady MHD viscous and incompressible fluid flow past a vertical plate is analyzed. The fluid is a gray, absorbing-emitting but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The governing equations are solved using runge-kutta fourth order method. Numerical results for the transient velocity, the temperature, the concentration, the local as well as average skin-friction, the rate of heat and mass transfer are shown graphically. It is observed that an increase in the magnetic field leads to decrease in the velocity field. The numerical predications have been compared with the existing information in the literature and good agreement is obtained.

Key Words: *Free Convection, MHD, Vertical Plate, Mass Transfer*.

INTRODUCTION

In astrophysical regimes, the presence of planetary debris, cosmic dust etc. creates a suspended porous medium saturated with plasma fluids. As in other porous media problems such as geo-mechanics and insulation engineering, the convectional approach is to simulate the pressure drop across the porous regime using Darcy linear model. Chamkha et al. [1] studied the radiation effects on the free convection flow past a semi-infinite vertical plate with mass transfer Muthucumaraswamy and Kumar Senthil [2] investigated the heat and mass transfer effect on moving vertical plate in the presence of thermal radiation. Choudhary et al. [3] studied the radiation effects with simultaneous thermal and mass diffusion in MHD mixed convection flow from a vertical surface. Emad M. Aboeldahad and Gamal El-Din A Azzam [4] investigated the thermal radiation effects on MHD flow past a semi-infinite vertical plate in the presence of mass diffusion.

The effects of chemical reaction and thermal stratification over a vertical stretching surface in a porous medium were considered by Mansour et al. [5]. Rashidi et al. [6] investigated analytically a steady, incompressible and laminar-free convective flow of a two-dimensional electrically conducting visco-elastic fluid over a moving stretching surface through a porous medium.

Heat and mass transfer in MHD flow over a permeable surface in the presence of slip is investigated by Turkyilmazoglu with two different thermal boundary conditions (PST and PHF) analytically [7]. Rashidi et al. [8] applied MHD flow in medicine science. They studied the dual control mechanisms of transverse magnetic field and porous media filtration in a buoyancy-driven blood flow regime in a vertical pipe, as a model of a blood separation configuration. Beg et al. [9] used Keller-Box implicit method to study heat and mass transfer micropolar fluid flow from an isothermal sphere. Turkyilmazoglu [10,11] presented multiple solutions in viscoelastic MHD fluid flow and heat and mass transfer over stretching and shrinking surfaces. Heat and mass transfer characteristics of MHD natural convective flow over a permeable, inclined surface with power-law variation of both wall temperature and concentration in presence of viscous dissipation and Ohmic heating are analyzed numerically by Chen [12]. Prasad et al. [13] studied a heat and mass transfer problem numerically in a non-Darcy porous medium over a vertical plate by Kellerbox method.

Satapathy et al. [14] studied the natural convection heat transfer in a Darcian porous regime with Rosseland radiative flux effects. With regard to thermal radiation heat transfer flows in porous media Raptis and Singh [15] studied numerically the natural convection boundary layer flow past an impulsively started vertical plate in a Darcian porous medium. Abd El-Naby et al. [16] presented a finite difference solution of radiation effects on MHD unsteady free convection flow over a vertical porous plate. Md. Abdus Samad and Mohammad Mansur Rahman [17] studied the thermal radiation interaction with unsteady MHD flow past a vertical porous plate immersed in a porous medium. Salem [18] studied the radiation and mass transfer effects in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium.

In many chemical engineering processes, there occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial application viz., polymer production, manufacturing of ceramics or glassware and food processing. Muthucumaraswamy and Chandrakala [19] studied radiative heat and mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction. Afify [20] investigated the effect of radiation on free convective flow and mass transfer past a vertical isothermal cone surface with chemical reaction in the presence of a transverse magnetic field.

However, the interaction of radiation with mass transfer in a chemically reacting and electrically conducting fluid in a Darcy porous medium has received little attention. Hence, the present study is attempted.

MATHEMATICAL ANALYSIS

The scenario under investigation comprises an unsteady two-dimensional hydro-magnetic, laminar natural convection flow of a viscous, incompressible, radiating and chemically reacting fluid past an impulsively started vertical plate embedded in a porous medium is considered. The x-axis is taken along the plate in the upward direction and the y-axis is taken normal to it. At time $t' = 0$ the plate commences impulsive motion in the x-direction, with constant velocity u_0, and the plate temperature and concentration levels are instantaneously elevated and are maintained constantly thereafter. Then, under the Boussinesq's approximation, the boundary layer equations for mass, momentum, energy and species conversation are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \dots (1)$$

$$\frac{\partial u}{\partial t'} + u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial y^2} + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2 u}{\rho} - \frac{\nu u}{k} \qquad \dots (2)$$

$$\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad \ldots (3)$$

$$\frac{\partial C}{\partial t'} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} - k_l C \quad \ldots (4)$$

In the foregoing equations, u and v are the velocity components along the x and y axes, ρ is density, μ is the magnetic permeability, σ is the electrical conductivity of the fluid, $\alpha = k/\rho c_p$ is the thermal diffusivity, k is the thermal conductivity, c_p is the specific heat at a constant pressure, D_m is the coefficient of mass diffusivity, T is the temperature, C is the fluid concentration and g is the acceleration due to gravity, β is the coefficient of volume expansion and β^* is the volumetric coefficient of expansion with concentration, k_l is the chemical reaction parameter.
The appropriate boundary conditions for the above problem are as follows

$t' \le 0$: $u = 0$, $v = 0$, $T = T_\infty$, $C = C_\infty$
$t' > 0$: $u = u_0$, $v = 0$, $T = T_w$, $C = C_w$ at $y = 0$
$u = 0$, $T = T_\infty$, $C = C_\infty$ at $x = 0$
$u \to 0$, $T \to T_\infty$, $C \to C_\infty$ at $y \to \infty$ … (5)

By using the Rosseland approximation (Brewster [21]), the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e}\frac{\partial T^4}{\partial y} \quad \ldots (6)$$

where σ_s is the Stefan-Boltzman constant and k_e is the mean absorption coefficient. It should be noted that by using the Rosseland approximation the present analysis is limited optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (6) can be linearized by expanding T^4 into the Taylor series about T_∞, which after neglecting higher order terms takes the form

$$T^4 \approx 4\, T_\infty^3\, T - 3\, T_\infty^4 \quad \ldots (7)$$

In view of equations (6) and (7), equation (3) reduces to

$$\frac{\partial T}{\partial t'} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma_s T_\infty^3}{3k_e \rho c_p}\frac{\partial^2 T}{\partial y^2} \quad \ldots (8)$$

Using the following non-dimensional quantities

$$X = \frac{xu_0}{\nu},\ Y = \frac{yu_0}{\nu},\ t = \frac{t'u_0^2}{\nu},\ U = \frac{u}{u_0},$$

$$V = \frac{v}{u_0},\ Gr = \frac{\nu g\beta(T_w - T_\infty)}{u_0^3},\ \mathrm{T} = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gc = \frac{\nu g\beta^*(C_w - C_\infty)}{u_0^3},\quad N = \frac{k_e k}{4\sigma_s T_\infty^3},\ Sc = \frac{\nu}{D}$$

$$C = \frac{C-C_\infty}{C_w-C_\infty}, \quad \Pr = \frac{\nu}{\alpha}, \; M = \frac{\sigma B_0^2 \nu}{u_0^2},$$

$$K = \frac{k_l \nu}{u_0^2}, \; Re = \frac{u_0 L}{\nu}, \; Da = \frac{k}{L^2} \qquad \ldots (9)$$

Equations (1), (2), (3) and (4) are reduced to the following non-dimensional form

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad \ldots (10)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = Gr\,T + Gc\,C + \frac{\partial^2 U}{\partial Y^2} - \left(M + \frac{1}{Da Re^2}\right)U \qquad \ldots (11)$$

$$\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{1}{\Pr}\left(1 + \frac{4}{3N}\right)\frac{\partial^2 T}{\partial Y^2} \qquad \ldots (12)$$

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - K\,C \qquad \ldots (13)$$

The corresponding initial and boundary conditions are

$t \le 0$: $U = 0$, $V = 0$, $T = 0$, $C = 0$

$t > 0$: $U = 1$, $V = 0$, $T = 1$, $C = 1$ at $Y = 0$

$U = 0$, $T = 0$, $C = 0$ at $X = 0$

$U \to 0$, $T \to 0$, $C \to 0$ at $Y \to \infty$ $\qquad \ldots (14)$

NUMERICAL TECHNIQUE

In order to solve the unsteady, non-linear, coupled equations (10) – (13), under the boundary conditions (14), an Runge–Kutta fourth order method has been employed.

The region of integration is considered as a rectangle with sides X_{max}(=1) and Y_{max}(=14),where Y_{max} corresponds to $Y = \infty$, which lies very well outside the momentum, energy and concentration boundary layers. The maximum of Y was chosen as 14 after some preliminary investigations so that the last two of the boundary condition of equations (14) are satisfied. The region to be examined in (X, Y, t) space is covered by a rectilinear grid with sides parallel to axes with ΔX, ΔY and Δt as the grid spacing in X, Y and t directions, respectively. An appropriate mesh sizes considered for the calculations are ΔX = 0.05, ΔY = 0.25 and the time step Δt = 0.01. The finite difference equations corresponding to the equations (10) – (13) are considered. During any one time step, the coefficients appearing in finite difference equations are treated as constants. The finite difference equations at every internal nodal point on a particular i-level constitute a tri-diagonal system of equations, which are solved by Thomas algorithm as discussed in Carnahan *et al.* [22]. Computations are carried out until the steady state is reached. The steady-state solution is assumed to have been reached, when the absolute difference between the values of velocity, temperature and concentration at two consecutive time steps is less than 10^{-5} at all grid points.

The derivates involved in equations (15) – (17) are evaluate using five-point approximation formula and then the integrals are evaluated using Newton-Cotes closed integration formula.

The truncation error in the finite difference approximation is $O(\Delta t^2+\Delta Y^2+\Delta X)$ and is tends to zero as Δt, ΔY and ΔX tends to zero. Hence the scheme is compatible. The finite difference scheme is unconditionally stable. Stability and compatibility ensure convergence.

RESULTS AND DISCUSSIONS

In order to get a physical insight of the problem, extensive computations have been performed for the effects of the controlling thermo fluid and hydrodynamic parameters on dimensionless velocity, temperature and concentration and also on the local and average skin friction, local and average Nusselt number and local Sherwood and average number.

The transient velocity profiles for different values of Gr, Gc, M and Da at a particular time t = 1.0 has been shown in Figure 1, An increase in Gr or Gc induces an increase in the velocity profiles. Darcy number stimulates the effect of bulk matrix impedance due to porous medium fibers. A rise in Da (which implies a rise in permeability, k) enhances the velocity of the fluid. There is a fall in transient velocity with the increase in M, the Lorentz force, which opposes the flow, also increases and leads to enhanced deceleration of the flow.

In Figure 2, the transient and steady state velocity profiles are presented for different values of N, Sc and K. The time required to reach the steady state and the velocity decreases with an increase in Sc or K.

Figure 3, shows that temperature decreases with an increase in values of Gr or Gc. It can also be seen that the time required to reach the steady state temperature is more at higher values of N(=10), as compared to lower values of N(=5). From Figures 2 and 3, it is noticed that with an increase in N, the velocity and temperature decrease accompanied by simultaneous reductions in both momentum and thermal boundary layers.

The transient concentration profiles for different values of Sc, K and N are shown in Figure 4, It is observed that for small values of Sc = 0.6, K= 0.5 and N = 5, the time required to reach the steady state is 8.10, whereas when N = 10, under similar conditions, the time required to reach the steady state is 8.31, from which it is concluded that for higher values of N, the time taken to reach the steady state is more when Sc is small. It is also observed that, increasing values of N corresponds to a thicker concentration boundary layer relative to the momentum boundary layer. Hence, it can be noted that at larger Sc, the time required to reach the steady state is less as compared to that of lower values of Sc. Also, an increase in Sc or K leads to a fall in the concentration.

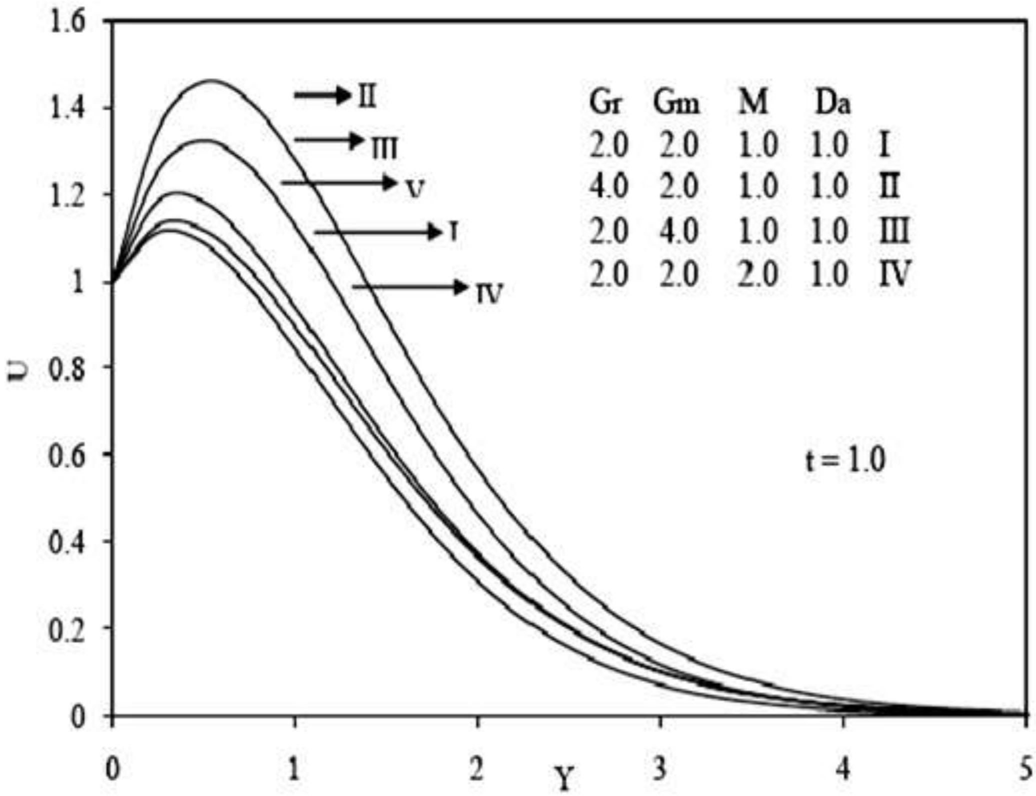

Fig 1 Transient Velocity Profiles at X=0 for different Gr and Gc

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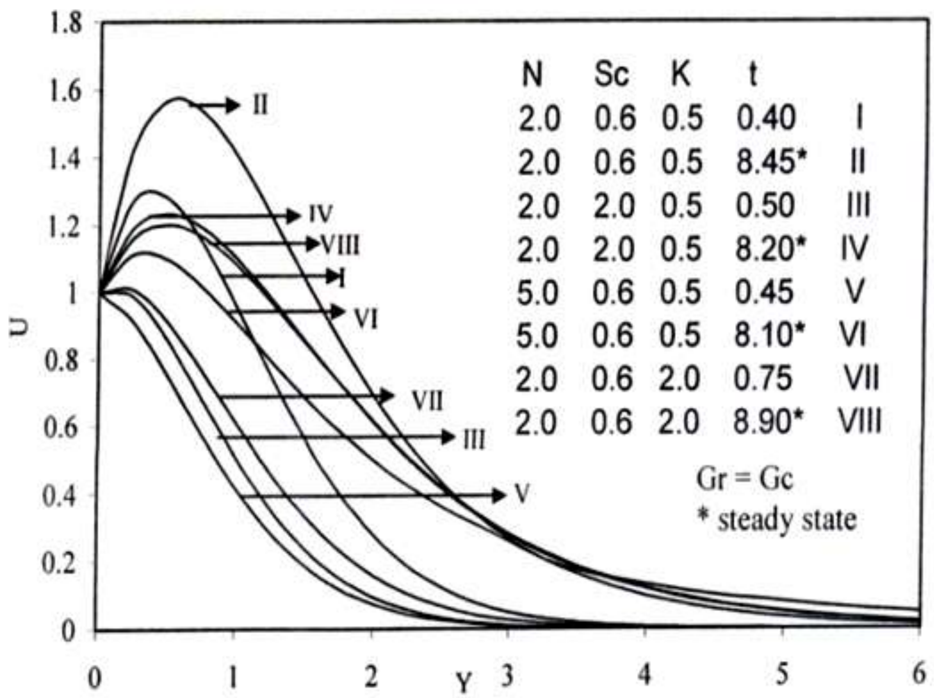

Fig 2 Velocity Profiles at X=1.0 for different N, Sc and K

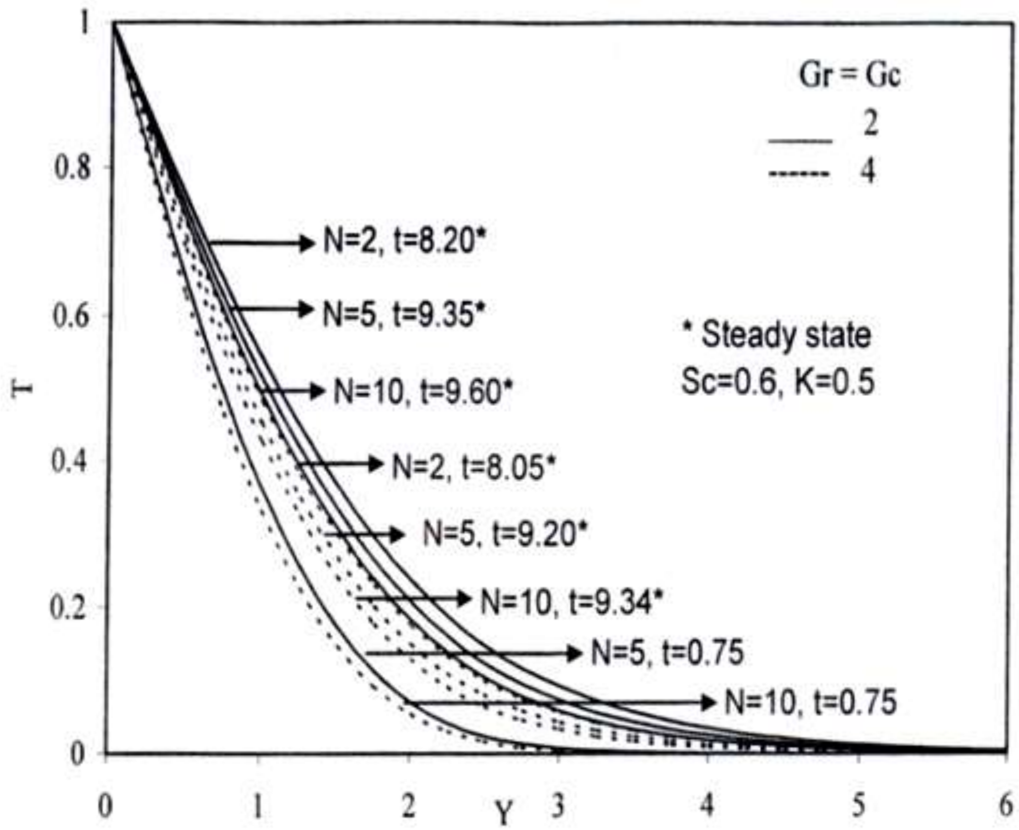

Fig 3 Temperature Profiles at X=1.0 for different Gr, Gc, N,

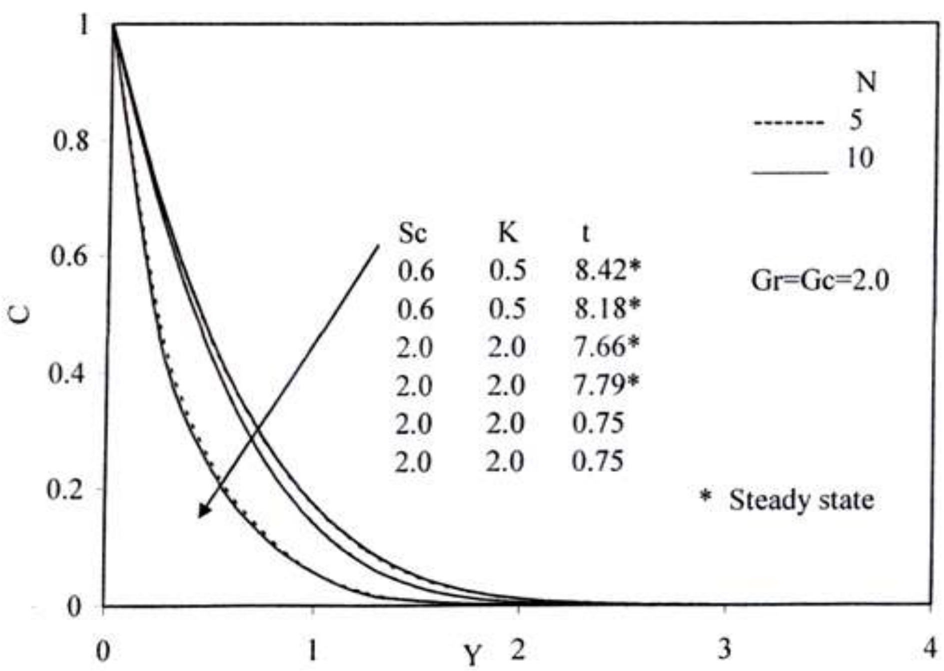

Fig 4 Concentration Profiles at X=1.0 for different Sc and K

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